

# A quantum solution to the arrow-of-time dilemma: Reply

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I acknowledge a flaw in the paper “A quantum solution to the arrow of time dilemma”: as pointed out by Jennings and Rudolph, (classical) mutual information is not an appropriate measure of information. This can be traced back to the quantum description underlying my analysis, where quantum mutual information is the appropriate measure of information. The core argument of my paper (summarized in its abstract) is not affected by this flaw. Nonetheless, I point out that such argument may not be adequate to account for all phenomena: it seems necessary to separately postulate a low entropy initial state.

The comment by Jennings and Rudolph [1] provides a clever example where (classical) mutual information (CMI) may increase even in situations where quantum mutual information (QMI) decreases. This is not unexpected as the QMI is only an upper bound to the CMI, as is well known and also stated in my paper [2]. However, their analysis does point out a flaw in my argument: I had implicitly assumed that a decrease of QMI between two systems always derives from the nullification of the QMI among two of the degrees of freedom in the two systems, which would indeed entail a decrease of CMI. I acknowledge that this hidden assumption is unwarranted.

The main claims contained in my paper (summarized in its abstract) are, however, not affected by this: the argument still goes through, although it is now clear that CMI is not the appropriate measure of information here. This is not too surprising, as in the quantum framework that is employed in my paper it is the QMI that measures the total amount of correlations (quantum *and* classical) [3]. Namely, the quantitative analysis in [2] should end with Eq. (2): a decrease of entropy of a system that does not follow from dumping entropy to an environment must entail a decrease of the *quantum* mutual information between the system and its observer.

It might be useful to reiterate the argument of [2]. Any information an observer has on a system must be encoded into some correlation between the system and a degree of freedom related to the observer. If one considers also the purification space (we assume that the universe is in a pure state, so such purification always exists), then this correlation consists of entanglement between the systems involved: the only correlations in a pure state arise from entanglement. If one of the systems is an observer (all observers agree with each other when they witness the same events [4]), then this entanglement entails from the observer’s point of view a probability stemming from the Born rule, and thus an associated entropy. The reduction of this entropy is in one-to-one correspondence with the reduction of the entanglement (as long as entropy is not transferred to the environment). This then entails a reduction of the information [8].

Only in quantum mechanics, correlation in a pure state is due to entanglement which in turn is responsible for the

entropy of a subsystem of such a system. Thus correlation and entropy are in one-to-one correspondence. This is not true in classical mechanics, where correlations can be increased at will without affecting the entropy. The fact that the variation in mutual information (a measure of correlation [3]) is equal to the local variations of von Neumann entropy (which can be put in one-to-one correspondence with thermodynamic entropy through Szilard engines or Maxwell demons), whenever the environment entropy is unaffected, is not an empty statement, contrary to Jennings and Rudolph’s claims: for example, an analogous statement is not true in classical mechanics.

In the decoherence language [5], my argument entails that, in order for a recoherence (namely a fusion of different Everett branches [4]) to happen from the point of view of the observer, all entanglement that built up during the decoherence event must be eliminated. This decorrelates the subsystems so that they are oblivious of having previously interacted during the decoherence process: no information on that interaction must be left (as it would prevent recoherence). Namely, any recoherence cannot leave trace of the previous decoherence.

One last important point which was unfortunately not addressed in my paper [2] must be emphasized. A good solution to the arrow-of-time dilemma must explain two main features: why entropy only increases and why it was so low in the initial state of the universe. My argument provides an answer to both: entropy only increases because any decrease cannot leave information of it having happened, and thus looking further into the past all observers will subjectively (but collectively) see entropy steadily decreasing for decreasing time up to a low initial entropy. However, my argument fails to describe the present. In fact, without any prior assumption on the initial entropy, one must assume that the universe is in a highly-probable high-entropy state [6] at every time (though it subjectively does not appear so when looking to the past). Then, any observer should see a high entropy state in his/her/its present. This is not what we observe: the current state of the universe has very high entropy, but is far from thermal equilibrium.

This implies that my argument is not a complete solution for the arrow-of-time dilemma: it seems that a

low entropy initial state (more precisely, a pure state where most subsystems were factorized and highly symmetric [7]) must be separately postulated to account for phenomena we see. My argument does, however, give a description of the second law from a quantum observer's point of view, which subjectively sees entropy constantly increasing even in extreme situations of a near-thermal equilibrium universe.

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- [8] Note that, as Jennings and Rudolph's example clearly shows, an interaction between system and environment can change the QMI between system and observer without affecting the observer's entropy or state.